# Algorithms Design and Analysis [ETCS-301]

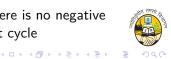
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### Bellman-Ford algorithm

- A graph with |V| vertices can have maximum |V|-1 edges and |V| vertices in any simple path.
- ▶ If number of edges in the path is greater than |V| 1 or vertices on the path is greater than |V| including source and destination then there is cycle in path.
- ightharpoonup Bellman works upon this fundamental and relaxes all edges |V|-1 times to find the shortest path from single source.
- After |V| 1 passes, there will be shortest path among nodes form source s, if exists.
- ▶ One more cycle of relax, if decrease the shortest path from source then it means there will be |V| edges and |V|+1 vertices and hence a negative weight cycle.
- ► The algorithm returns either TRUE if there is no negative weight cycle or FALSE if negative weight cycle



#### Algorithm

#### BELLMAN-FORD(G,w,s)

- 1. INITIALIZE-SINGLE-SOURCE(G, s)
- 2. for i = 1 to |V| 1
- 3. for each edge  $(u, v) \in E$
- 4. RELAX(u, v, w)
- 5. for each edge  $(u, v) \in E$
- 6. if d[v] > d[u] + w(u, v)
- return FALSE
- 8. return TRUE

Time complexity of the algorithm is O(VE)



## Correctness of the Bellman-Ford algorithm I

Statement: If G contains no negative-weight cycles that are reachable from s, then the algorithm returns TRUE, we have  $d[v] = \delta(s,v)$  for all vertices  $v \in V$ , and the predecessor subgraph  $G_{\pi}$  is a shortest-paths tree rooted at s. If G does contain a negative-weight cycle reachable from s, then the algorithm returns FALSE.

- ► Suppose that graph *G* contains no negative-weight cycles that are reachable from the source *s*.
- Now we have to prove that the algorithms will return TRUE.
- If vertex v is reachable from s, then at termination after |V|-1 iterations  $d[v]=\delta(s,v)$
- If vertex v is unreachable from s, then at termination after |V|-1 iterations  $d[v]=\delta(s,v)=\infty$



### Correctness of the Bellman-Ford algorithm II

After termination of |V|-1 iterations, for all edge  $(u,v) \in E, d[v] = \delta(s,v)$ 

$$d[v]=\delta(s,v)$$
  $d[u]=\delta(s,u)$   $d[v]=\delta(s,v)\leq \delta(s,u)+w(u,v)$ -by the triangle inequality  $d[v]< d[u]+w(u,v)$ 

- ▶ So  $d[v] \le d[u] + w(u, v)$  for all edge  $(u, v) \in E$  in the predecessor subgraph  $G_{\pi}$  is a shortest path from source s
- ▶ So d[v] > d[u] + w(u, v) of line no ?? of the algorithm will never TRUE and hence algorithm will return TRUE.
- ► Suppose that graph *G* contains negative-weight cycles that are reachable from the source *s*.



## Correctness of the Bellman-Ford algorithm III

- Now we have to prove that the algorithms will return FALSE.
- Let  $c = \langle v_0, v_1, \dots, v_k \rangle$  is a negative weight cycle in the path p reachable from source s in graph G where  $v_0 = v_k$
- $ightharpoonup \sum_{i=1}^{n} w(v_{i-1}, v_i) < 0 \leftarrow \text{because of negative weight cycle}$
- Now let contradiction that there is a negative weight cycle but still algorithm returns TRUE.
- ► So  $d[v_i] \le d[v_{i-1}] + w(v_{i-1}, v_i)$  for i = 1, 2, ..., k.
- ► Taking summation over all the k values

$$\sum_{i=1}^k d[v_i] \leq \sum_{i=1}^k d[v_{i-1}] + \sum_{i=1}^k w(v_{i-1}, v_i)$$





# Correctness of the Bellman-Ford algorithm IV

 $d[v_1] + d[v_2] + \cdots + d[v_{k-1}] + d[v_k]$ 

$$\leq d[v_0] + d[v_1] + \cdots + d[v_{k-1}] + \sum_{i=1}^k w(v_{i-1}, v_i)$$

- $\qquad \qquad 0 \leq \sum_{i=1}^k w(v_{i-1}, v_i), \because v_0 = v_k$
- ▶ But  $\sum_{i=1}^{n} w(v_{i-1}, v_i) < 0$  because of negative weight cycle
- So our assumption of contradiction is wrong and the algorithm will return FALSE.





#### Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91~9911375598

