Module 1: Assignment 1

January 4, 2022

1. What are the possible binary operations on sets? Explain with the help of suitable examples.

2. Obtain truth table for:

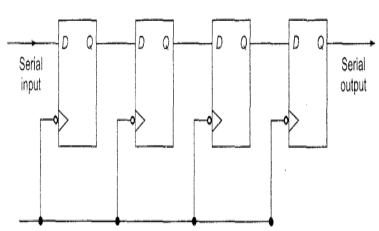
   - $\alpha = (P \vee Q) \wedge (P \implies Q) \wedge (Q \implies P)$
   - $\alpha = (P \vee Q) \implies ((P \vee R) \implies (R \vee Q))$

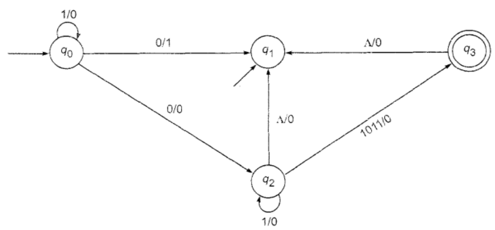
3. Explain the following for logical identities with the help of suitable examples:

   - Absorption Laws
   - De-morgan laws
   - Double Negation laws

4. What do you understand by the term directed graph? Explain using suitable example considering terms such as end vertices, predecessor, successor, degree, path, connected graph and circuit.

5. Consider a 4-bit serial shift register as a finite state machine.





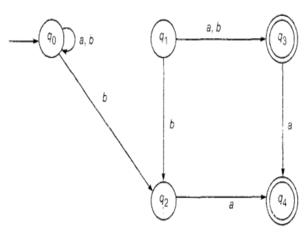
State	Input a	Input b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3		q_2

6. Consider the given transition system: Determine the initial states, final states and acceptability of 101011, 111010.

7. Prove that for any transition function δ and for any two input string $'x'$ and $'y'$
   $\delta(q, xy)=\delta(\delta(q, x), y)$

8. Construct a deterministic finite automaton equivalent to
   M=($\{q_0, q_1, q_2, q_3\}$,{a,b},δ,q_0,$\{q_3\}$)
   where δ is as under:

9. Construct a DFA equivalent to NDFA 'M' whose transition diagram is given as:

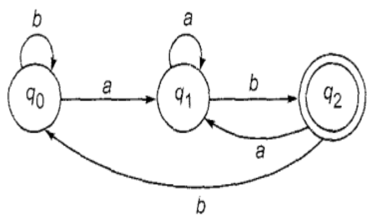


10. Construct a DFA equivalent to NDFA with initial state q_0 whose transition table is defined as

11. Construct a DFA accepting all strings over {a,b} ending in *ab*.

12. Construct a DFA equivalent to NDFA for:

State	a	b
q_0	q_1, q_3	q_2, q_3
q_1	q_1	q_3
q_2	q_3	q_2
q_3	–	–



13. M=($\{q_1, q_2, q_3\}$,{0,1}, δ,q_1,$\{q_3\}$) is a NDFA where δ is given by:
$\delta(q_1, 0)$=$\{q_2, q_3\}$
$\delta(q_1, 1)$=$\{q_1\}$
$\delta(q_2, 0)$=$\{q_1, q_2\}$
$\delta(q_2, 1)$=ϕ
$\delta(q_3, 0)$=$\{q_2\}$
$\delta(q_3, 1)$=$\{q_1, q_2\}$
Construct equivalent DFA.

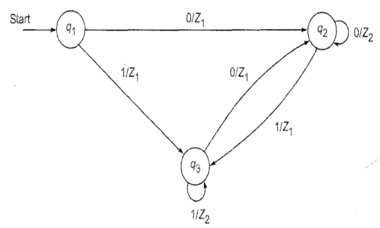
14. Convert the given mealy machine into equivalent moore machine

15. Construct minimum state automaton equivalent to given automata M:

16. Construct a grammar G so that L(G) = $\{a^n b a^m$ |n, m $\geq$ 1}

17. If G is S $\rightarrow$ aS |a, then show that L(G)=$\{a\}^+$

State	Input		
	0	1	$\wedge$
$\rightarrow q_0$	q_0, q_3	q_0, q_1	
q_1	q_2		
q_2	q_2	q_2	q_4
q_3	q_4		
q_4	q_4	q_4	



State $\backslash\Sigma$	a	b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
q_6	q_1	q_3

18. If the grammar G is given by the productions S $\rightarrow$ aSa |bSb |aa |bb |$\wedge$, show that:
    - L(G) has no strings of odd length
    - Any string in L(G) is of length 2n, n $\geq$ 0
    - the number of strings of length 2n is 2^n
19. Find the highest type number which can be applied to the following productions:
    - S $\rightarrow$ Aa, A $\rightarrow$ c |Ba, B $\rightarrow$ abc
    - S $\rightarrow$ ASB |d, A $\rightarrow$ aA
    - S $\rightarrow$ aS |ab
20. Differentiate between Recursive Set and Recursively Enumerable Set
21. Prove that Context-sensitive language is recursive.
22. Prove that there exists a recursive set which is not a contxt-sensitive language over {0,1}.
23. Let G=({A,B,S},{0,1},P.S) where P consists of S $\rightarrow$ 0AB, A0 $\rightarrow$ S0B, A1 $\rightarrow$SB1, B $\rightarrow$SA, B $\rightarrow$01. Show that L(G)= ϕ.

24. Find the language generated by grammar S $\rightarrow$ AB, A $\rightarrow$A1 |0, B $\rightarrow$ 2B |3. Can the above language be generated by a grammar of higher type?

25. Construct a grammar which generates all even integer upto 998.

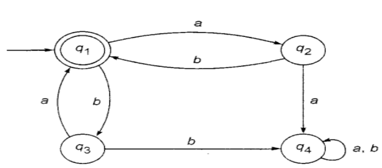
26. Construct CFG to generate the following:

- $\{0^m 1^n$ |m$\neq$n, m,n $\geq$1$\}$
- $\{a^l b^m c^n$|one of l,m,n equals 1 and remaining tqo are equal$\}$
- $\{a^l b^m c^n$ |l+m=n$\}$
- The set of all strings over {0,1} containing twice as many 0's and 1's

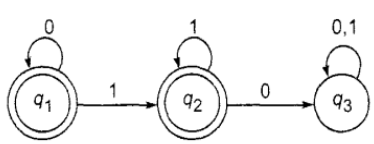
27. Show that G_1=({S},{a,b},P_1,S) where P_1={S $\rightarrow$ aSb |ab} is equivalent to G_2=({S,A,B,C},{a,b},P_2,S), where P_2 consists of S $\rightarrow$AC, C$\rightarrow$SB, S $\rightarrow$AB, A$\rightarrow$a,B $\rightarrow$b

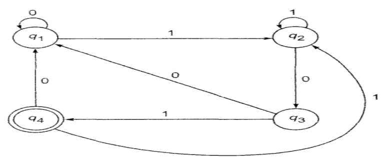
28. What are the applications of different grammar types?

29. Prove that the finite automaton whose transition diagram below accepts the set of all strings over alphabet {a,b} with an equal number of **a's** and **b's**, such that each prefix has atmost has atmost one more **a** than the **b's** and atmost one more **b** than the **a's**



30. Describe in English the set accepted by finite automaton whose transition diagram is as under:





State/ Σ	0	1
$\rightarrow q_0$	(q_0,R)	(q_1, R)
q_1	(q_1,R)	(q_2, L)
q_2	(q_0,R)	(q_2, L)

31. Construct a regular expression corresponding to the state diagram described as under:

32. Give R.E. for representing the set L of strings in which every 0 is immediately followed by atleast two 1's. Prove that R.E. r= $\wedge + 1^*(011)^*(1^*(011)^*)^*$ also describes the same set of strings.

33. Prove $(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1) = 0^*1(0 + 10^*1)^*$

34. Determine the acceptability of 101001 for the following: where Q=$\{q_0, q_1, q_2\}$, s=q_0, t=q_1, r=q_2

35. Construct DFA with reduced states equivalent to R.E. (10+(0+11)0*1).

36. Construct transition system equivalent to R.E.

    - $(ab + c^*)^*b$
    - $a + bb + bab^*a$
    - $(a + b)^*abb$

37. Prove that
    $(a^*ab + ba)^*a^* = (a + ab + ba)^*$

38. Construct a finite automata accepting all strings over {0,1} ending in 010 or 0010.

39. Construct a regular grammar which can generate the set of all strings starting with a letter (A to Z) followed by a string of letters or digits (0 to 9).

40. Show that L=$\{0^i 1^i \mid i \geq 1\}$ is not regular.

41. Show that L= $\{ww \mid w \in \{a, b\}^* \}$ is not regular.

42. Is L = $\{a^{2n} \mid n \geq 1\}$ regular ?