Algorithms Design and Analysis [ETCS-301]

Dr. A K Yadav
Amity School of Engineering and Technology
(affiliated to GGSIPU, Delhi)
akyadav1@amity.edu
akyadav@akyadav.in
www.akyadav.in
+91 9911375598

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Medians and Order Statistics

- The i^{th} **Order Statistic** of a set of n elements is the i^{th} smallest element.
- The 1^{st} **Order Statistic** is the **minimum** of the set.
- The n^{th} **Order Statistic** is the **maximum** of the set.
- **Median** is the middle element of the set.
- If n is odd then unique median will be at $(n+1)/2$
- If n is even then two median **lower** and **upper** will be at $n/2$ and $n/2+1$ respectively.
- So **lower median** will be at $\lfloor (n+1)/2 \rfloor$ and **upper median** will be at $\lceil (n+1)/2 \rceil$ for both n even or odd.

Minimum and Maximum I

MINIMUM(A,n)

1 $min = A[1]$
2 $Loc = 1$
3 for $i = 2$ to n
4 if $min > A[i]$
5 $min = A[i]$
6 $Loc = i$
7 return Loc

Number of comparison is $n - 1$

Minimum and Maximum II

Min-Max(A,n)

1. $min = A[1]$
2. $MinLoc = 1$
3. $max = A[1]$
4. $MaxLoc = 1$
5. for $i = 2$ to n
6. if $min > A[i]$
7. $min = A[i]$
8. $MinLoc = i$
9. if $max < A[i]$
10. $max = A[i]$
11. $MaxLoc = i$
12. return $MinLoc, MaxLoc$

Number of comparison is $2(n-1)$

We can reduce this to $3\left\lfloor \frac{n}{2} \right\rfloor$

Minimum and Maximum III

Min-Max(A,n)

1. if $n \leq 0$ return
2. if $n\%2 = 0$ and $n \geq 2$
3. if $A[1] < A[2]$
4. $Min = A[1]$
5. $MinLoc = 1$
6. $Max = A[2]$
7. $MaxLoc = 2$
8. else
9. $Min = A[2]$
10. $MinLoc = 2$
11. $Max = A[1]$
12. $MaxLoc = 1$

Minimum and Maximum IV

13. for $i = 3$ to n with step size 2
14. if $A[i] < A[i+1]$
15. if $A[i] < min$
16. $min = A[i]$
17. $MinLoc = i$
18. if $A[i+1] > max$
19. $max = A[i+1]$
20. $MaxLoc = i+1$
21. else
22. if $A[i+1] < min$
23. $min = A[i+1]$
24. $MinLoc = i+1$
25. if $A[i] > max$

Minimum and Maximum V

26. $max = A[i]$
27. $MaxLoc = i$
28. else
29. $Min = A[1]$
30. $MinLoc = 1$
31. $Max = A[1]$
32. $MaxLoc = 1$
33. for $i = 2$ to n with step size 2
34. if $A[i] < A[i + 1]$
35. if $A[i] < min$
36. $min = A[i]$
37. $MinLoc = i$
38. if $A[i + 1] > max$

Minimum and Maximum VI

39. $max = A[i+1]$
40. $MaxLoc = i+1$
41. else
42. if $A[i+1] < min$
43. $min = A[i+1]$
44. $MinLoc = i+1$
45. if $A[i] > max$
46. $max = A[i]$
47. $MaxLoc = i$
48. return $MaxLoc, MinLoc$

Number of comparison is:

$$\begin{cases} 3\frac{n-1}{2} & \text{if n is odd} \\ 3\frac{n-2}{2}+1 & \text{if n is even} \end{cases}$$

or we can say max number of comparison is $3\left\lfloor \frac{n}{2} \right\rfloor$

Minimum and Maximum VII

RANDOMIZED-SELECT(A,p,r,i)

1. if $p == r$
2. return $A[p]$
3. $q = RANDOMIZED - PARTITION(A, p, r)$
4. $k = q - p + 1$
5. if $i == k$
6. return $A[q]$
7. elseif $i < k$
8. return $RANDOMIZED - SELECT(A, p, q - 1, i)$
9. else
10. return $RANDOMIZED - SELECT(A, q + 1, r, i - k)$

Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91 9911375598