Algorithms Design and Analysis [ETCS-301]

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Minimum Spanning Trees (MST) I

- Tree is a connected undirected acyclic graph.
- Tree is a set of nodes and set of edges connected that nodes that is $T = (V, E)$ and each edge $(u, v) \in E$ having weight $w(u, v)$
- Minimum means the sum of all edges to connect all nodes is minimum
- Spanning means tree span over all the nodes.
- So Minimum Spanning Tree is a undirected weighted acyclic graphs that covers all the nodes with minimum sum of the weights.
- There will be $n - 1$ edges for n vertices in the MST.

Minimum Spanning Trees (MST) II

- Let G is a undirected graph with set of vertices V and set of edges E and each edge $(u, v) \in E$ having weight $w(u, v)$. We have to find out an acyclic subset $T \subseteq G$ that connects all of the vertices and whose total weight is minimum that is

$$\text{minimize } W(T) = \sum_{(u,v) \in T} w(u, v)$$

- T in this case will be MST.

GENERIC-MST(G,w)

1. $A = \emptyset$
2. while A does not form a spanning tree
3.   find an edge (u, v) that is safe for A
4. $A \cup \{(u, v)\}$
5. return A

Kruskal's algorithm I

- Kruskal's algorithm uses the least weight edge to connect the two tree to make one.
- Kruskal's algorithm qualifies as a greedy algorithm because at each step it adds to the forest an edge of least possible weight
- It uses a disjoint-set data structure to maintain several disjoint sets of elements
- The operation $FIND - SET(u)$ returns address of the representative element from the set that contains u
- We check $FIND - SET(u)$ and $FIND - SET(v)$ for any edge (u, v). If they return the same address then it means the are already connected.
- If $FIND - SET(u)$ and $FIND - SET(v)$ returns different value that it means they are the members of two different tree and using $UNION(u, v)$ we make one tree connected by edge (u, v)

Kruskal's algorithm II

MST-KRUSKAL(G,w)

$A = \emptyset$
for each vertex $v \in V$
    $MAKE - SET(v)$
sort the edges of E into nondecreasing order by weight w
for each edge $(u, v) \in E$ taken in nondecreasing order by weight
    if $FIND - SET(u) \neq FIND - SET(v)$
        $A \cup \{(u, v)\}$
        $UNION(u, v)$
return A

Time complexity of the algorithm is $O(E \lg E)$ or $O(E \lg V)$ as $|E| < |V|^2 \Rightarrow \lg E = O(\lg V)$

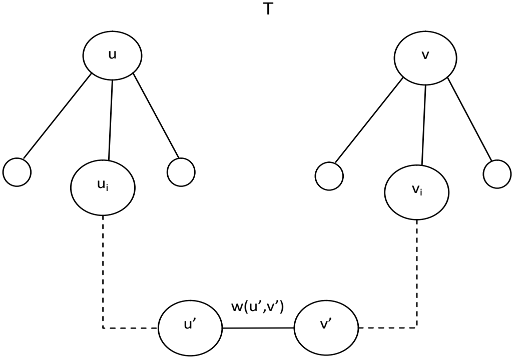
Correctness of Kruskal's Algorithm I

Prove that if $w(u, v)$ is minimum weighted edge of the graph G then edge (u, v) will be part of the some MST T.

- Suppose T is a MST with weight $W(T)$ and edge (u, v) is not the part of the tree T.
- It means u and v are connected by some other edges being tree T is spanning tree.

Correctness of Kruskal's Algorithm II

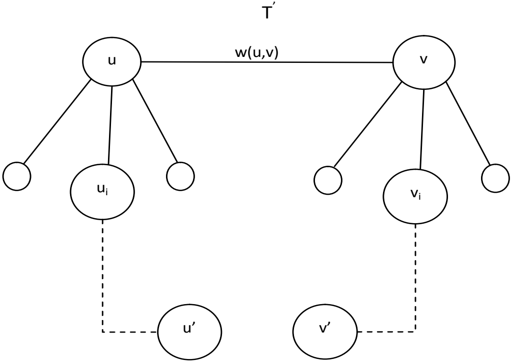
- Suppose vertex u is connected by u_i vertices and v is connected by v_j vertices as shown in below figure:



- Now suppose $(u^{'}, v^{'})$ is a least weight edge from the path $(u, u_i, \ldots, v_j, v)$

Correctness of Kruskal's Algorithm III

- Remove $(u^{'}, v^{'})$ and add edge (u, v) to make the tree connected as shown in below figure:



- The weight of the tree $T^{'}$ will be: $W(T^{'}) = W(T) -$ weight of the edge removed $+$ weight of the edge added

- $\Rightarrow W(T^{'}) = W(T) - w(u^{'}, v^{'}) + w(u, v)$
- $\Rightarrow W(T^{'}) = W(T) - (w(u^{'}, v^{'}) - w(u, v))$
- $\Rightarrow W(T) - W(T^{'}) = w(u^{'}, v^{'}) - w(u, v)$
- $\Rightarrow W(T) - W(T^{'}) \geq 0$
- $\Rightarrow W(T) \geq W(T^{'})$
- But $W(T)$ is the MST as per assumption so $W(T)$ can't be greater than $W(T^{'})$
- So $W(T) = W(T^{'})$ and hence $T^{'}$ is also a MST and (u, v) will be the part of MST $T^{'}$

Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91 9911375598