Algorithms Design and Analysis [ETCS-301]

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Examples

1. Show that $an + b = O(n^2)$
2. Show that $\frac{1}{2}n^2 - 3n = \Theta(n^2)$
3. Show that $\frac{1}{2}n^2 - 3n = \Omega(n)$

1. Show that $an + b = O(n^2)$
   we have to prove that $an + b \leq cn^2$ for some positive constants c, n_0 and for all $n \geq n_0$.

$$\begin{aligned} an + b &\leq an + |b| \text{ for all } n \geq 1 \\ &\leq an + |b|n \text{ for all } n \geq 1 \\ &= (a + |b|)n \\ &= cn \text{ where } c = a + |b| \\ &\leq cn^2 \end{aligned}$$

$$\Rightarrow an + b \leq cn^2 \text{ where } c = a + |b| > 0 \text{ all } n \geq n_0 = 1$$

$$\therefore an + b = O(n^2)$$

$$\Rightarrow an + b < cn^2 \text{ for all } c > 0 \text{ all } n > n_0 = max(1, -b/a)$$

$$\therefore an + b = o(n^2)$$

Solutions II

2. Show that $\frac{1}{2}n^2 - 3n = \Theta(n^2)$
We have to prove that $c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2$ for some positive constants c_1, c_2, n_0 and for all $n \geq n_0$.

$$c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2$$

$$\Rightarrow c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2 \text{ after dividing } n^2$$

$$\Rightarrow 0 < c_1 \leq \frac{1}{2} - \frac{3}{n}$$

$$\Rightarrow 0 < \frac{1}{2} - \frac{3}{n}$$

$$\Rightarrow \frac{3}{n} < \frac{1}{2}$$

Solutions III

$\Rightarrow n > 6$

Let $n = 7$

$\Rightarrow 0 < c_1 \leq \frac{1}{14}$

Let $c_1 = \frac{1}{14}$

Now $0 < \frac{1}{2} - \frac{3}{n} \leq c_2$

$\Rightarrow 0 < \frac{1}{2} \leq c_2$ when $n = \infty$

Let $c_2 = \frac{1}{2}$

$\Rightarrow 0 < \frac{1}{14}n^2 \leq \frac{1}{2}n^2 - 3n \leq \frac{1}{2}n^2$ for all $n \geq n_0 = 7$ and any constant $0 < c_1 \leq \frac{1}{14} \leq \frac{1}{2} \leq c_2$.

3. Show that $\frac{1}{2}n^2 - 3n = \Omega(n)$
   We have to prove that $cn \leq \frac{1}{2}n^2 - 3n$ for some positive constants c, n_0 and for all $n \geq n_0$.

Let $cn \leq \dfrac{1}{2}n^2 - 3n$

$c \leq \dfrac{1}{2}n - 3$ after dividing n both side

$\Rightarrow c \leq \dfrac{1}{2}n + 3$

$\Rightarrow c \leq \dfrac{1}{2}n + 3n$ for all $n > 0$

$\Rightarrow c \leq \dfrac{7}{2}n$

$$\Rightarrow \frac{7}{2} \leq \frac{7}{2}n \text{ for all } n \geq 1$$

$$\therefore c \leq \frac{7}{2}$$

$$\text{Let } c = \frac{7}{2}$$

$\Rightarrow \frac{7}{2}n \leq \frac{1}{2}n^2 - 3n$ for all positive constants $c \leq \frac{7}{2}$ and for all $n \geq n_0 = 1$.

Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91 9911375598