Algorithms Design and Analysis [ETCS-301]

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Binomial Coefficient

- $(x+a)^n$ can be expended as
  ${}^nC_0x^na^0 + {}^n C_1x^{n-1}a^1 + \cdots + {}^n C_kx^{n-k}a^k + \ldots {}^n C_nx^0a^n$
- Coefficient of the term $x^{n-k}a^k$ is know as Binomial Coefficient $C(n,k) = {}^n C_k$
- $C(n,k) = \frac{n!}{n-k!k!}$
- $C(n,k) = C(n-1,k-1) + C(n-1,k)$ if $n > k > 0$
- $\Rightarrow \frac{n-1!}{n-k!k-1!} + \frac{n-1!}{n-k-1!k!}$
- $\Rightarrow \frac{n-1!}{n-k!k-1!}\frac{k}{k} + \frac{n-1!}{n-k-1!k!}\frac{n-k}{n-k}$
- $\Rightarrow \frac{(n-1)!k}{n-k!k!} + \frac{(n-1)!(n-k)}{n-k!k!}$
- $\Rightarrow \frac{(n-1)!(k+n-k)}{n-k!k!}$
- $\Rightarrow \frac{n!}{n-k!k!}$
- $\Rightarrow C(n,k)$
- $C(n,0) = C(n,n) = 1$

Computing a Binomial Coefficient I

Bottom-up Approach:

No optimal solution only fixed value

BC(c,n,k)

1. Let $c[0 \ldots n, 0 \ldots k]$
2. for $i = 0$ to n
3. for $j = 0$ to i
4. if $i = j$ or $j = 0$
5. $c[i, j] = 1$
6. else
7. $c[i, j] = c[i - 1, j - 1] + c[i - 1, j]$
8. return c

Computing a Binomial Coefficient II

Top-down Approach:

Mamoized-BC(c,n,k)

1. Let $c[0 \ldots n, 0 \ldots k]$
2. for $i = 0$ to n
3. for $j = 0$ to i
4. $c[i, j] = 0$
5. return $BC(c, n, k)$

$BC(c, i, j)$

1. if $c[i, j] > 0$
2. return $c[i, j]$
3. if $i = j$ or $j = 0$
4. return 1
5. return $c[i, j] = BC(c, i - 1, j - 1) + BC(c, i - 1, j]$

Complexity of the Knapsack is $O(nk)$

Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91 9911375598