Algorithms Design and Analysis [ETCS-301]

Dr. A K Yadav
Amity School of Engineering and Technology
(affiliated to GGSIPU, Delhi)
akyadav1@amity.edu
akyadav@akyadav.in
www.akyadav.in
+91 9911375598

October 23, 2019

Dijkstra's algorithm I

- It is used to find single-source shortest-paths problem on a non-negative weighted,directed graph
- S is the set of vertices whose final shortest-path weights from the source s have already been determined
- The algorithm repeatedly selects the vertex $u \in V - S$ with the minimum shortest-path estimate
- Adds u to S, and relaxes all edges leaving u.
- We use a min-priority queue Q of vertices based on their d values

Algorithm: DIJKSTRA(G,w,s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. $S = \emptyset$
3. $Q = V$
4. while $Q \neq \emptyset$

Dijkstra's algorithm II

5. $u =$EXTRACT-MIN(Q)
6. $S = S \cup \{u\}$
7. for each vertex $v \in adj[u]$
8. $\quad RELAX(u, v, w)$

Time complexity of the algorithm is $O(V \lg V + E \lg V)$ or $O(E \lg V)$ using Min-heap and $O(E + V \lg V)$ using Fibonacci heap.

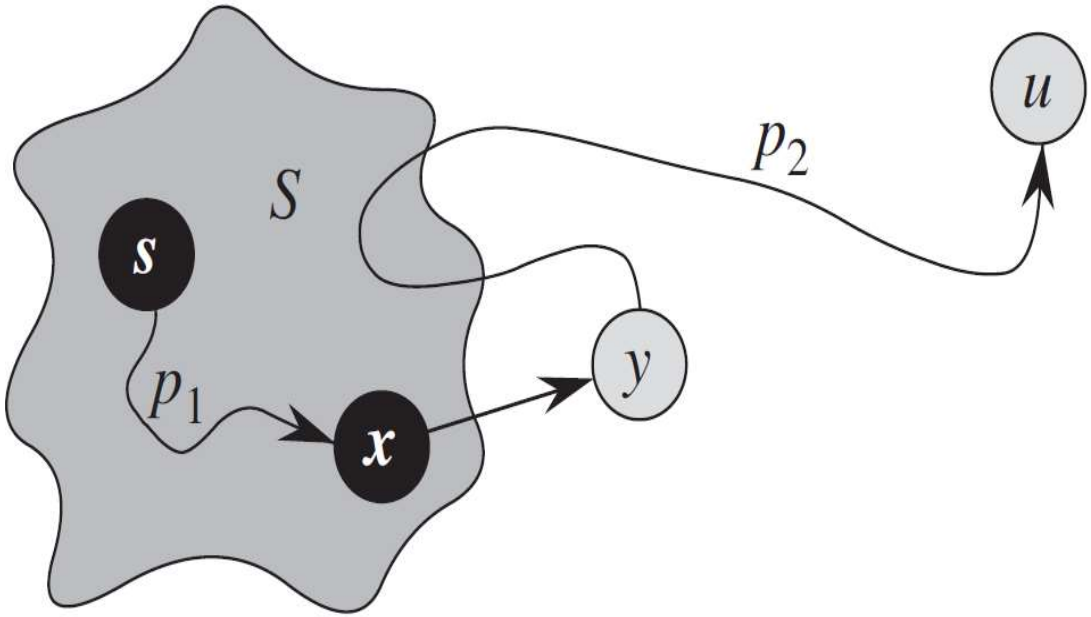
Correctness of Dijkstra's algorithm I

We have to prove that $d[u] = \delta(s, u)$ when u is added to set S for each vertex $u \in E$ if u is reachable from s.

- We will prove it by contradict.
- Let u is the first vertex such that $\delta(s, u) < d[u] < \infty$ when add to S.
- $u \neq s$ because s is the first vertex added to the S and $d[s] = \delta(s, s) = 0$
- $S \neq \emptyset$ because at least s was there in S before u is added to S

Correctness of Dijkstra's algorithm II

- There must be at least one path from s to u because our assumption is that $\delta(s, u) < d[u] < \infty$, There must be at least one shortest path p from s to u.



- Let p is a path from s to u before adding u into S such that $s \rightsquigarrow x \rightsquigarrow y \rightsquigarrow u$ and $s, x \in S$ and $y, u \notin S$

Correctness of Dijkstra's algorithm III

- $d[y] = \delta(s, y)$ because y is predecessor of u so it will be relax before u and as per assumption u is the first vertex where $d[u] \neq \delta(s, u)$
- $\delta(s, y) \leq \delta(s, u)$ because y is the predecessor of u and at the the shortest path from s also there is no negative weights so $w(y, u) \geq 0$

$$d[y] = \delta(s, y)$$

$$\leq \delta(s, u)$$

$$\leq d[u]$$

- But because both vertices $y, u \notin S$ and when we extract u from Q then $d[u] \leq d[y]$
- $d[y] \leq d[u]$ and $d[u] \leq d[y]$ is only possible when $d[u] = d[y]$

Correctness of Dijkstra's algorithm IV

- So $d[y] = \delta(s, y) = \delta(s, u) = d[u]$ means out assumption is wrong.
- Hence proved that $d[u] = \delta(s, u)$ when u is added to set S

Thank you

Please send your feedback or any queries to akyadav1@amity.edu, akyadav@akyadav.in or contact me on +91 9911375598